LATTICES OF ORDER IDEALS AS MONOIDAL INTERVALS

MICHAEL PINSKER
HITOTSUBASHI UNIVERSITY, TOKYO
JAPAN

The clone lattice on an infinite set X can be naturally partitioned into so-called monoidal intervals, that is, sets of clones which have the same unary functions. We approach the question what monoidal intervals can look like, and show that the situation is quite diverse: If $\mathfrak{L}$ is the lattice of order ideals of some partial order $\mathfrak{P}$ with smallest element such that $|\mathfrak{P}| \leq 2^{|X|}$, then there is a monoidal interval in the clone lattice on X which is isomorphic to $\mathfrak{L}$. In particular, we find that if $\mathfrak{L}$ is any chain with smallest element which is an algebraic lattice, then $1 + \mathfrak{L}$ appears as a monoidal interval; also, if $Y \subseteq X$, then the power set of Y with an additional smallest element is isomorphic to a monoidal interval. Concerning possible cardinalities of monoidal intervals these results imply that there are monoidal intervals of all cardinalities smaller than $2^{|X|}$, as well as monoidal intervals of cardinality 2^{λ}, for all $\lambda \leq 2^{|X|}$.